Homework Assignment 7

COT3100 – Spring 2021

1. **(21)** Let set 𝐴 = {1,2,3,4} and let 𝑅1 and 𝑅2 be binary relations on 𝐴. Specifically, let:

𝑅1 = {(1,1), (1, 2), (2, 1), (2,2), (2, 4), (3, 4), (4, 2), (4, 3) (4,4)} 𝑅2 = {(1,2), (1, 3), (1, 4),(2,1), (2, 3), (4, 1), (4, 2)}

Determine the following:

* + 1. Whether 𝑅1 is reflexive, irreflexive, symmetric, anti-symmetric and/or transitive.

Lacks so is **not reflexive**, contains at least one therefore **not irreflexive**. Contains and for every point (1,2,3,4) therefore **is symmetric** and is **not anti-symmetric**. is not transitive therefore **not transitive**.

* + 1. Whether 𝑅2 is reflexive, irreflexive, symmetric, anti-symmetric and/or transitive.

Lacks any so **is irreflexive, not reflexive**. Only two points, are symmetric therefore is **not symmetric** and is **not anti-symmetric**. Lacks as relates to but there is no **not** **is transitive**.

* + 1. 𝑅1 ∘ 𝑅2.

* + 1. 𝑅2 ∘ 𝑅1.

* + 1. 𝑅1 ∪ 𝑅2.

f) 𝑅1 ∩ 𝑅2.

g) The reflexive, symmetric and transitive closures of both 𝑅1 and 𝑅2. (14 pairs->r1 12 pairs ->r2)

1. **(10)** Let 𝑅 be a binary relation over the positive integers defined as follows:

𝑅 = {(𝑎, 𝑏) | 2𝑏 < 𝑎 < 3𝑏 }

Determine whether 𝑅 satisfies the following properties. Give brief justifications for your answers.

* + - 1. reflexive

can never be equal to because can never be less than [ doesn’t work]. For example, doesn’t work because is false. Therefore, it **isn’t reflexive**

* + - 1. irreflexive

Since there can never be any reflexive pairs **it must be irreflexive**

* + - 1. symmetric

can never be smaller than [ must be true for a possible combination] or the equation doesn’t work and therefore there can never be a symmetric pair. For example, works but does not. Therefore, it **is not symmetric**

* + - 1. anti-symmetric

Because there cannot be any symmetric or reflexive pairs therefore **it must be anti-symmetric**

* + - 1. transitive

are both true howeveris not and therefore **is not transitive**

1. **(10)** Let 𝐴 = {1,2, 3, 4, 5,6, 7, 8}. How many possible symmetric relations over 𝐴 contain the ordered pairs

(2, 3), (3, 2), (4, 7), (5, 5) and (8, 7)?

1. **(9)** Let 𝑓 and 𝑔(𝑥) = 𝑥2.
   1. (6) Determine ℎ1(𝑥) = 𝑓(𝑔(𝑥)) and ℎ2(𝑥) = 𝑔(𝑓(𝑥)).

* 1. (3) What are the largest possible domains for which ℎ1 and ℎ2 can be defined?